Supplementary Information

for **Dynamic Adsorption Layer and Flow within Liquid Meniscus in Forced Dewetting**

V.I. Kovalchuk, G.K. Auernhammer

Effect of inertia in wedge flow

By using the velocity distribution for the case of zero viscous stresses at the solution surface, Eqs. (9)-(10), one can estimate the ratio of the inertial to viscous terms in the equation for radial velocity component as

$$Q_m = \frac{\rho U r}{\mu} \frac{(\varphi \cos\varphi \sin(\varphi-\theta) - (\varphi-\theta)\sin\varphi \cos(\varphi-\theta))\sin(\varphi-\theta)}{(\varphi - \cos\varphi \sin\varphi)\cos(\varphi-\theta)} \qquad \text{(S1)}$$

For the second case of zero velocity at the solution surface, Eqs. (11)-(12), the ratio of the inertial to viscous terms in the equation for radial velocity component can be estimated as

$$Q_{im} = \frac{\rho U r}{\mu} \frac{(\varphi(\varphi-\theta)\sin\theta - \theta \sin\varphi \sin(\varphi-\theta))(\sin\varphi \cos(\varphi-\theta) - \varphi\cos\theta)}{(\varphi^2 - \sin^2\varphi)(\sin\varphi \sin(\varphi-\theta) + \varphi \sin\theta)} \qquad \text{(S2)}$$

The inertial terms in the equations for angular velocity components appear to be zero in the both cases. The angular dependencies of the ratios Q_m and Q_{im}, given by Eqs. (S1) and (S2), are shown in Fig. S1.

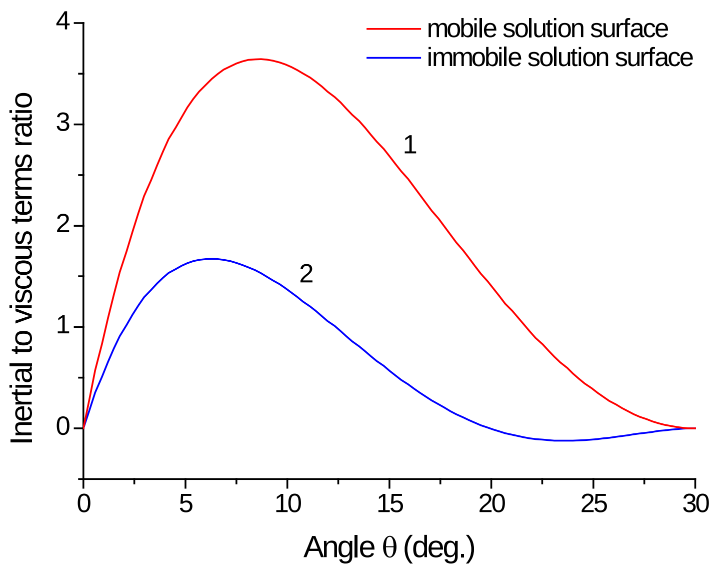

Fig. S1. Variation of the inertial to viscous terms ratio with the angle θ in a wedge flow, according to Eqs. (S1) and (S2), for a mobile (1) and immobile (2) solution surface at U = 10 cm/s, r = 1 mm, $\mu/\rho = 10^{-6}$ m^2/s, and φ = 30°.

The ratios Q_m and Q_{im} are proportional to the multiplier $\rho Ur/\mu$, which for $U = 0.1$ m/s, $r = 10^{-3}$ m, and $\mu/\rho = 10^{-6}$ m^2/s is equal to 100. However, due to the small angular parts in Eqs. (S1) and (S2), the inertial to viscous terms ratios appear to be much smaller: the average ratio is of about 2 for a mobile and of about 0.7 for an immobile solution surface, as it is seen from Fig. S1. And in the equation for angular velocity component the inertial terms are zero for all angles θ, as mentioned above.

Thus, the real contribution of the inertial terms to the velocity distribution for a wedge flow is much smaller than it could be expected from the large Reynolds number value ($Re = 100$ for $r = 1$ mm). The numerical solutions of the full Navier-Stockes equations confirmed a negligible contribution of the inertial terms for the chosen system parameters. An example, presented in Fig. S2, shows that the difference between the flow patterns in a plane wedge calculated for $Re = 1$ and 100 is insignificant.

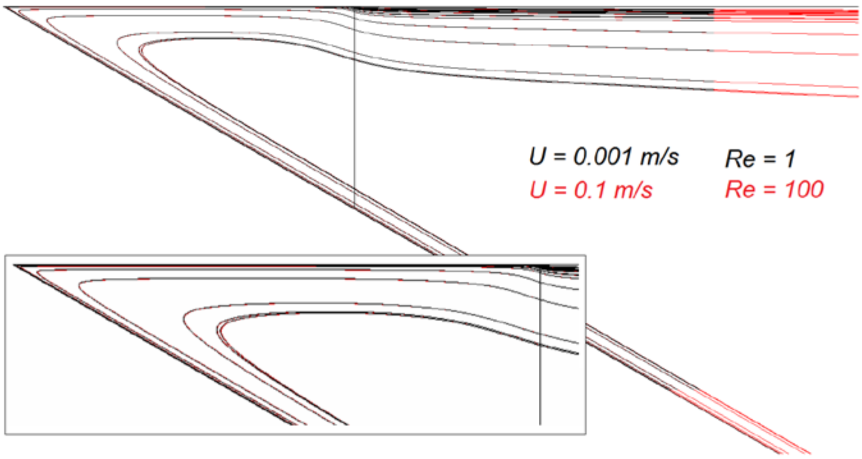

Fig. S2. Flow pattern in a plane wedge with one steadily moving flat surface and one surface composed of a mobile and an immobile parts, calculated for $Re = \rho Ur/\mu = 1$ (black lines) and 100 (red lines) at $r = 1$ mm with the same other parameters as in Fig. S1. The black vertical line marks the border between the mobile and immobile part of the solution surface at the distance 1 mm from the contact line. The inset is a zoom of the part closest to the contact line.

Eqs. (S1) and (S2) show that the contribution of the inertial terms turns to zero at both wedge surfaces (θ = 0 and θ = φ). The ratio of the inertial to viscous terms tend to zero for small wedge angles φ, when the flow structure becomes similar to a plain-parallel flow in a flat channel. This ratio increases with increasing angle φ and increasing distance from the contact line r. The difference between inertial and non-inertial flow behavior in a wedge was analyzed recently in [S1].

Lubrication approximation

If the contact angle φ is small, and the local thickness of the meniscus changes slowly with the distance from the contact line then the lubrication approximation can be used. Following de Gennes [S2], we can write the steady-state velocity profile near the *free part* of the surface as

$$u_x(y) = U\left[\frac{1}{2} - \frac{3}{2}\left(\frac{y}{h}\right)^2\right] \tag{S3}$$

where x and y are the tangential and normal coordinates at the solution surface, respectively, U is the substrate velocity, and $h = \varphi \cdot x$ is the local thickness of the meniscus ($0 \le y \le h$). The velocity profile, Eq. (S3), satisfies three necessary conditions:

- non-slip condition for the substrate surface $u_x(h) = -U$;
- zero viscous stresses at the free liquid surface $\left.\frac{\partial u_x}{\partial y}\right|_{y=0} = 0$;
- conservation of mass - zero total volume flow in each cross-section under steady flow conditions $\int_0^h u_x(y)dy = 0$.

According to Eq. (S3) the velocity at the free part of the surface is $u_x(0) = \frac{U}{2}$.

We will assume here that the surface is free for $0 < x < x_0$ and is retarded for $x > x_0$. For the *retarded part* of the surface we have the velocity profile given by the equation

$$u_x(y) = U\left[2\frac{y}{h} - 3\left(\frac{y}{h}\right)^2\right] \tag{S4}$$

It satisfies three necessary conditions:

- non-slip condition at the substrate surface $u_x(h) = -U$;
- zero velocity at the immobile part of the surface $u_x(0) = 0$;
- conservation of mass - zero total volume flow in each cross-section under steady flow conditions $\int_0^h u_x(y)dy = 0$.

According to Eq. (S4), the shear stress at the retarded part of the surface is non-zero: $\left.\frac{\partial u_x}{\partial y}\right|_{y=0} = \frac{2U}{h}$. It is easy to check, that the equations in this subsections are the limiting case of the respective Moffatt equations for φ << 1.

Near the point $x = x_0$ we have a sharp decrease of the surface velocity $u_s = u_x(0)$ from $U/2$ to zero and a sharp increase of the viscous stress from 0 to $2U/h$. There should be a transition zone, where the velocity profile transforms from Eq. (S3) to Eq. (S4). But in lubrication approximation the size of this transition zone is small - of the order of the local thickness of the liquid film $h_0 = \varphi \cdot x_0 << x_0$, for $\varphi << 1$, and we will ignore this transition zone.

For the retarded part of the surface we can write the stress balance as

$$\frac{d\gamma}{dx} = -\mu \left.\frac{du_x}{dy}\right|_{y=0} = -\frac{2\mu U}{h} \tag{S5}$$

where γ is the local surface tension and μ is the viscosity of the liquid. The local thickness varies with the distance as $h = \varphi \cdot x$, therefore one obtains a differential equation for the surface tension

$$\frac{d\gamma}{dx} = -\frac{2\mu U}{\varphi x} \tag{S6}$$

which can be easily integrated

$$\Pi = \gamma_0 - \gamma_{eq} = \frac{2\mu U}{\varphi} ln \frac{\lambda_C}{x_0} \tag{S7}$$

Here we used the conditions that the surface tension is equal to the surface tension of pure water, γ_0, at the leading edge of the retarded zone ($x = x_0$), and it is approximately equal to the equilibrium surface tension, γ_{eq}, on the distances of the order of capillary length $\lambda_C = \sqrt{\gamma_{eq}/\rho g}$.

From Eq. (S7) we find an approximate expression for the size of the free part of the meniscus surface as

$$x_0 = \lambda_C \, exp\left(-\frac{\Pi\varphi}{2\mu U}\right) \tag{S8}$$

[S1] Mahmood A, Siddiqui AM: **Two Dimensional Inertial Flow of a Viscous Fluid in a Corner**. *Appl Math Sci* 2017, **11**: 407- 424. https://doi.org/10.12988/ams.2017.612282

[S2] de Gennes PG: **Deposition of Langmuir-Blodgett layers**. *Colloid Polymer Sci* 1986, **264**: 463-465. https://doi.org/10.1007/BF01419552